CMB Lensing Power Spectrum without Noise Bias

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Stanford Cosmology Seminar — April 1, 2024

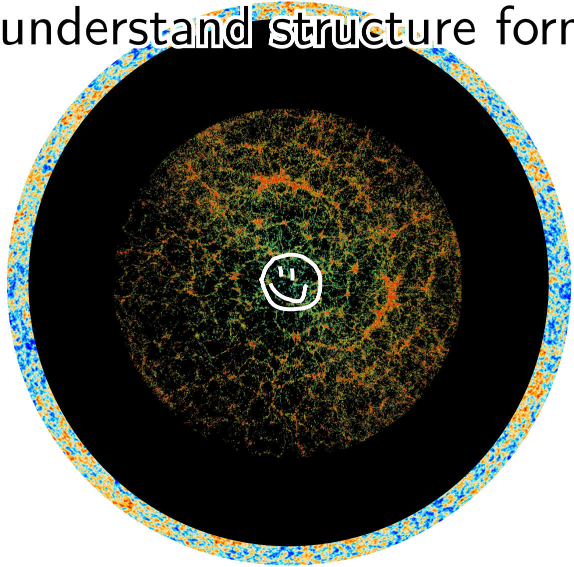
arxiv:2402.04309 with Manu Schaan and Simone Ferraro

Initial density perturbation in matter collapses gravitationally to form cosmic structure

The dynamics of structure formation (e.g. how clumpy matter is over time) carries a lot of information about physics (Σm_ν, *nature of dark energy, properties of dark matter, . . .*)

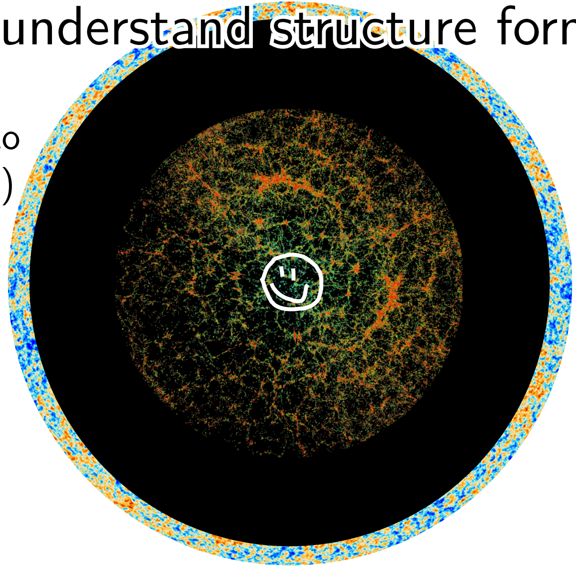
Illustris

Do we understand structure formation?



Do we understand structure formation?

(1) Fit ΛCDM to CMB ($z \approx 1100$) then predict clumpyness of matter today (indirect)



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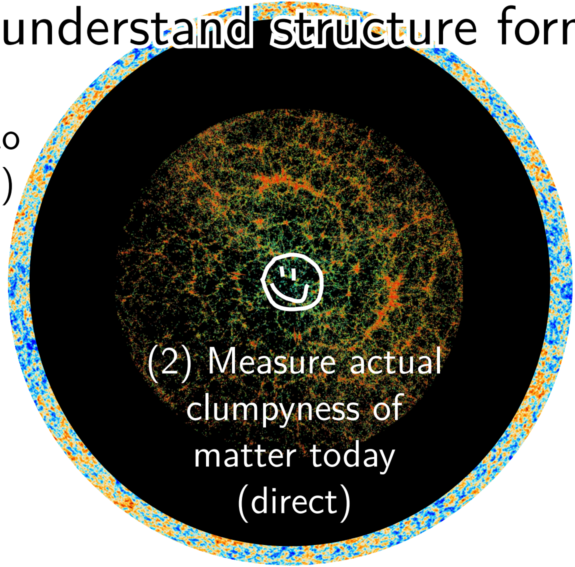
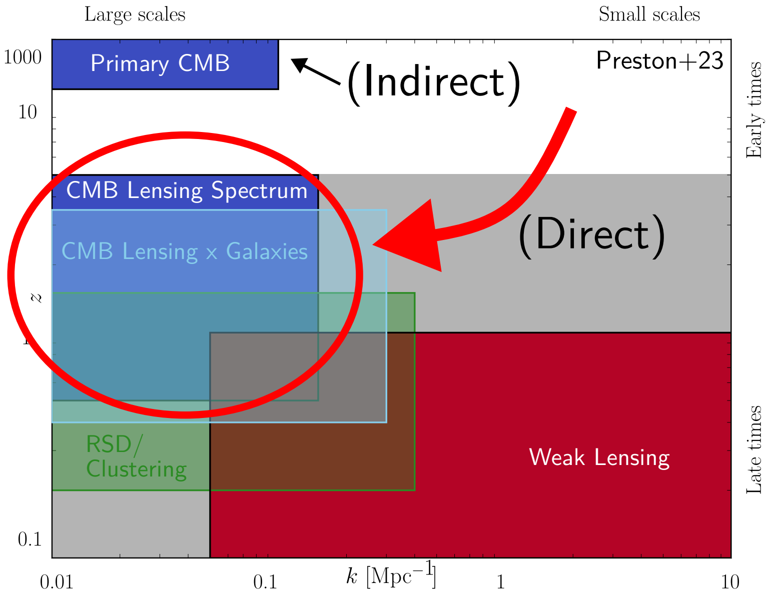
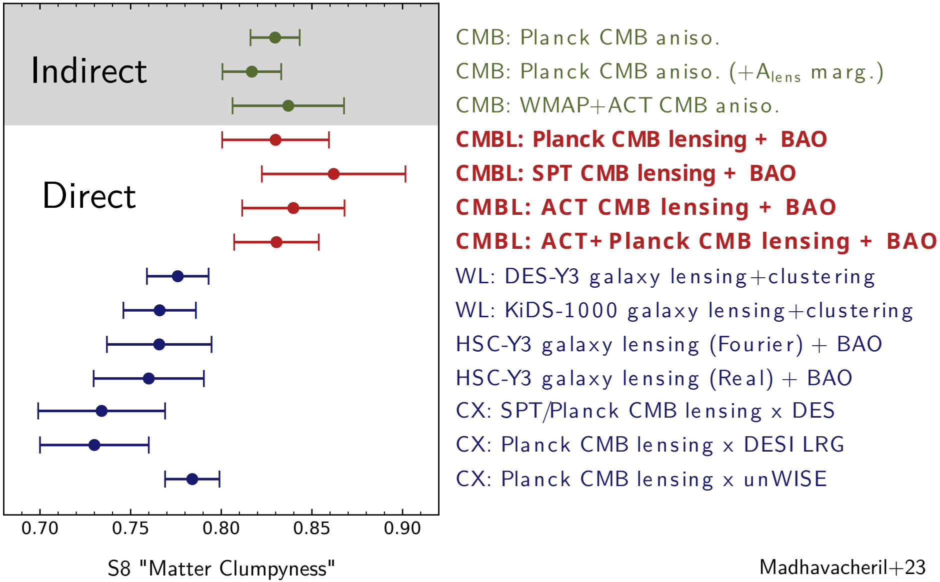
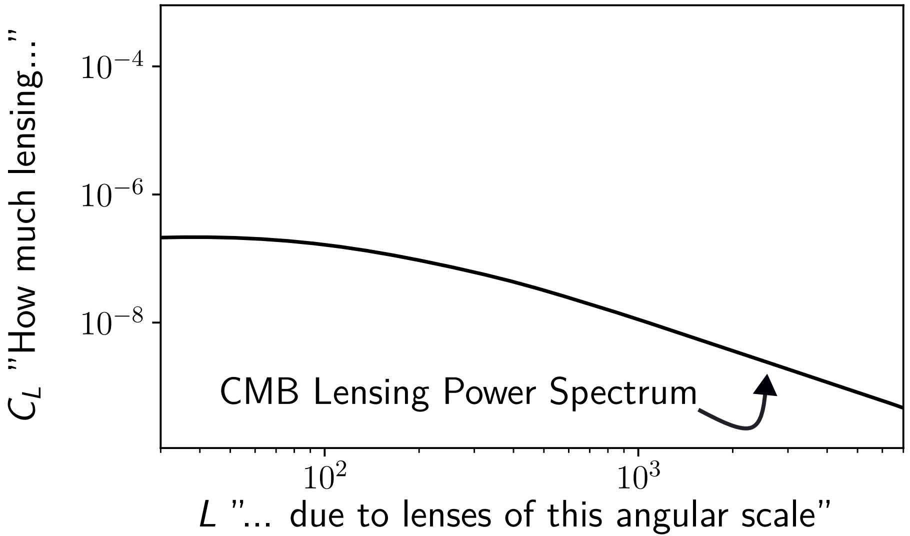


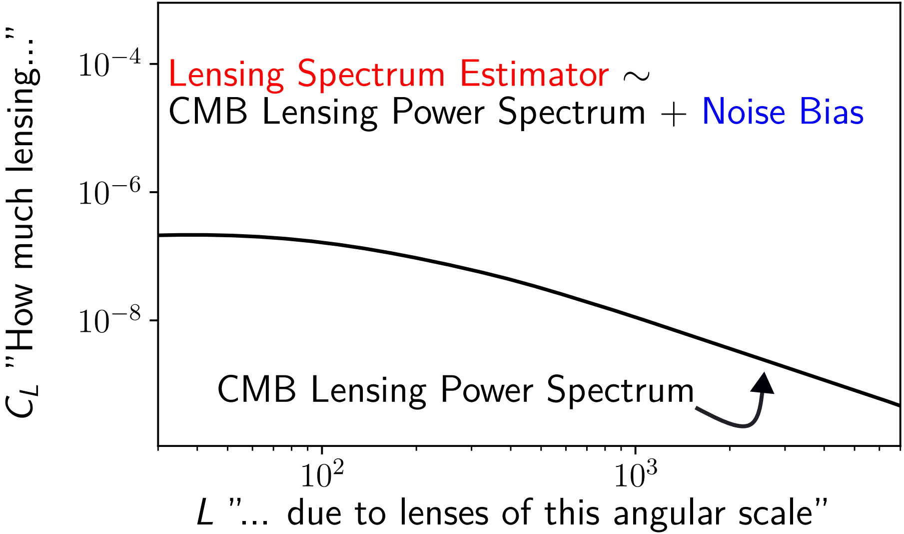
CMB lensing probes different scales and redshifts from other direct probes!





Madhavacheril+23


C_L "How much lensing..."
10^{-4}
10^{-6}
10^{-8}
CMB Lensing Power Spectrum
10^2
10^3
L "... due to lenses of this angular scale"


Lensing Spectrum Estimator ∼
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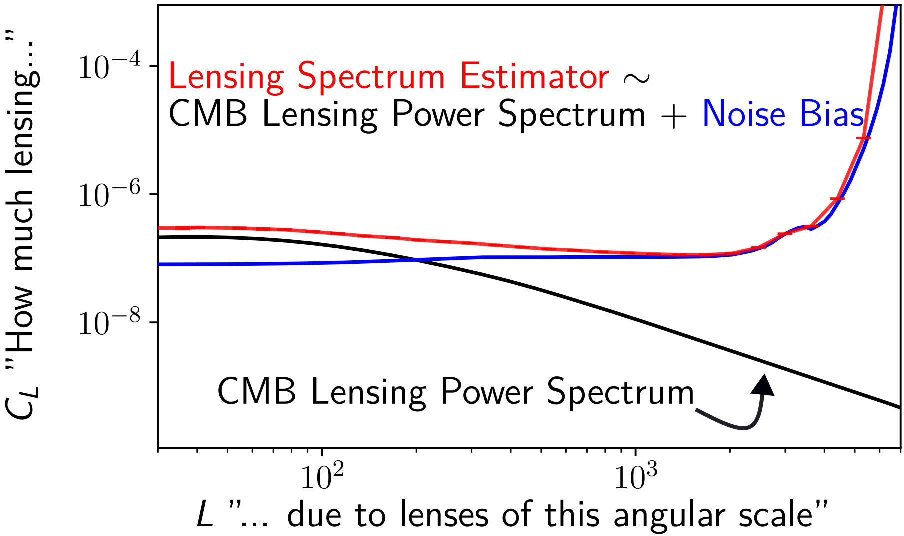

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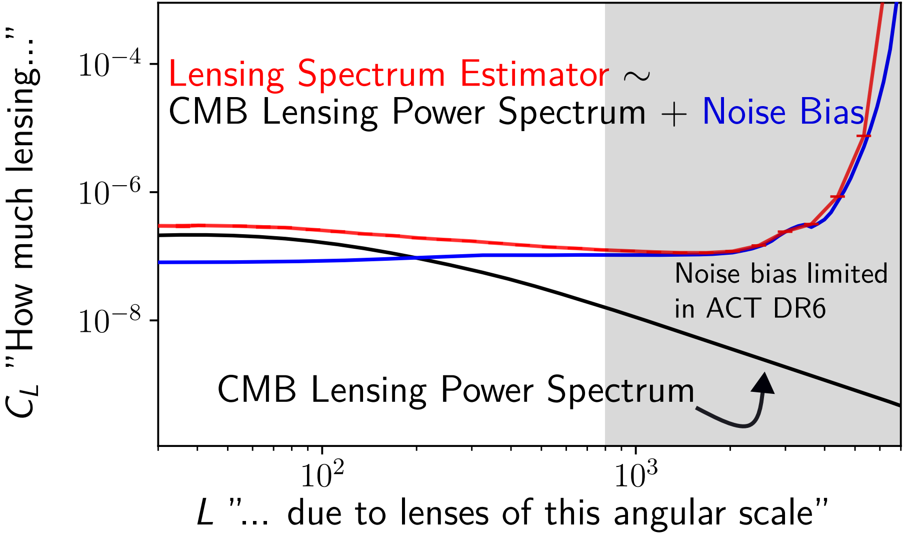
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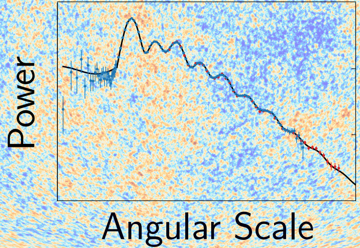
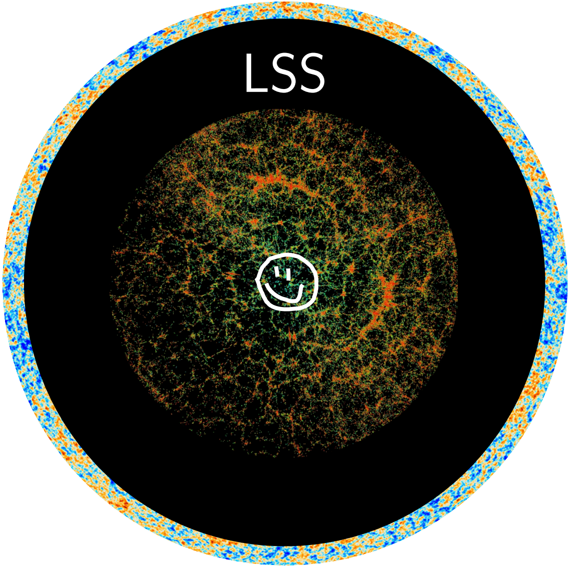
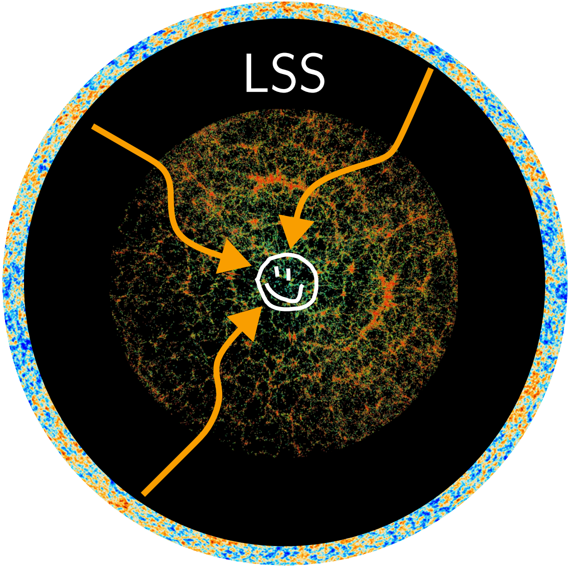
$T(\boldsymbol{\theta})$: Fluctuations

Statistically **Gaussian, Homogeneous, Isotropic**



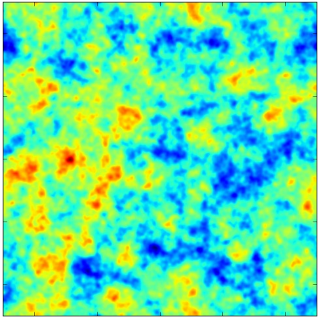


CMB



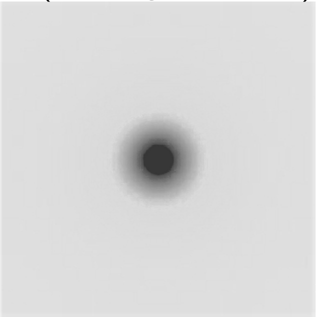
CMB
lensed by
LSS

Unlensed CMB



$+$

∇(Lensing Potential)



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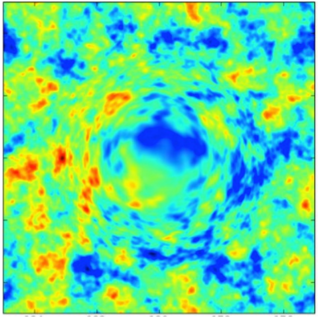
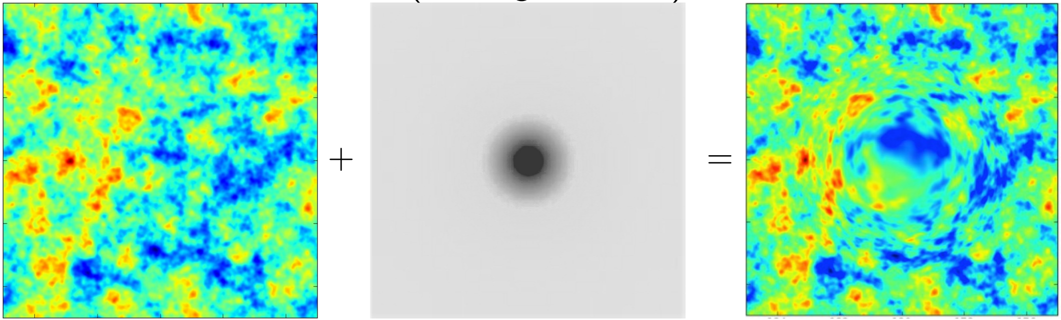
Lensed CMB



Sherwin

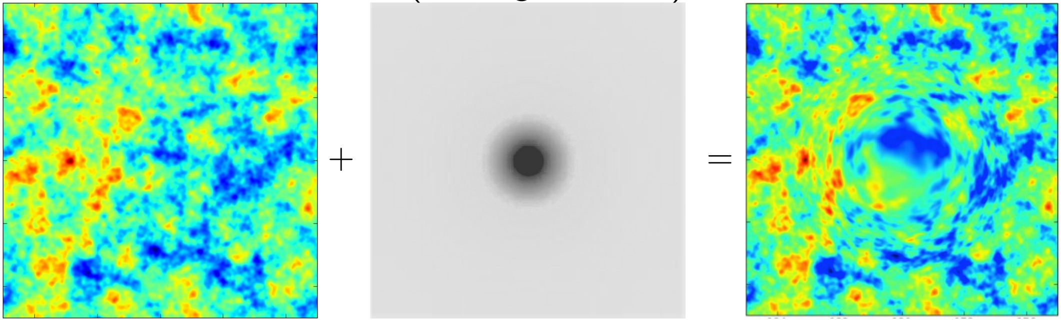
Unlensed CMB $\quad$ ∇(Lensing Potential) $\quad$ Lensed CMB



Sherwin

$$T^{\text{lensed}}(\hat{\boldsymbol{n}}) = T^{\text{unlensed}}(\hat{\boldsymbol{n}} + \nabla\{\text{Lensing Potential}(\hat{\boldsymbol{n}})\})$$

Unlensed CMB | ∇(Lensing Potential) | Lensed CMB



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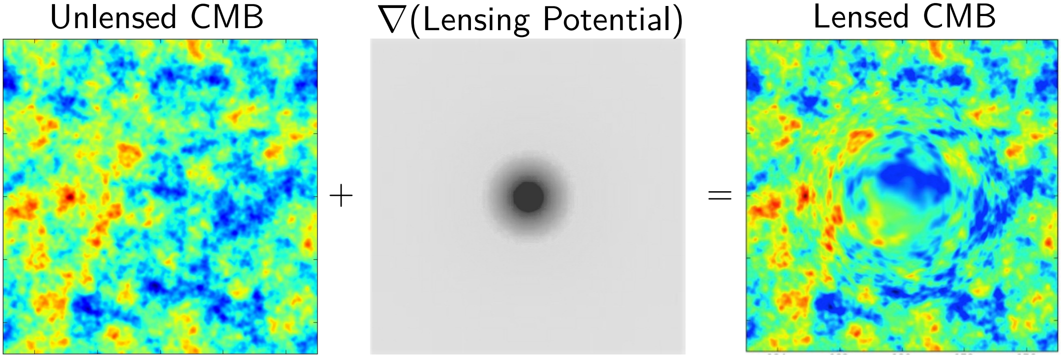

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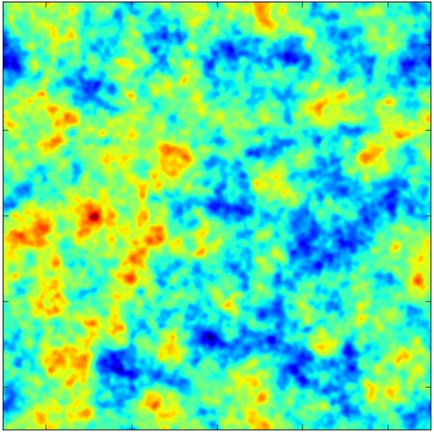

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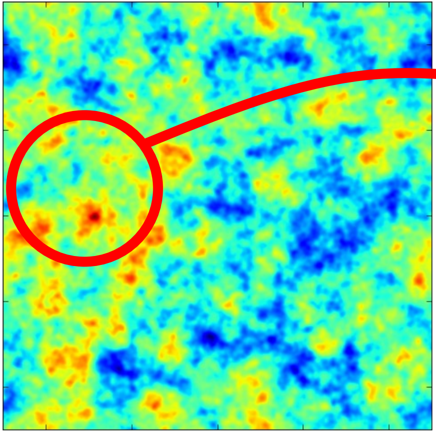

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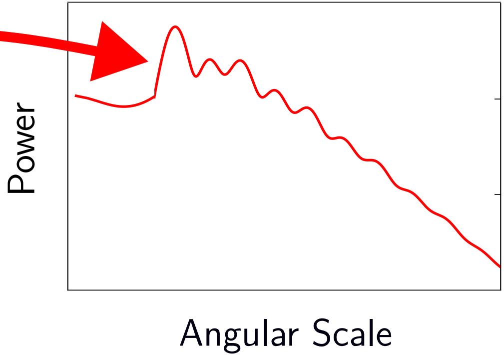

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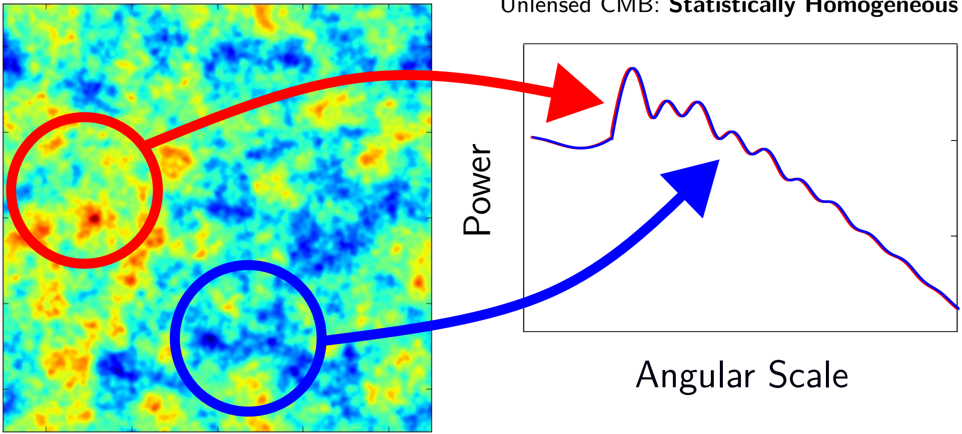

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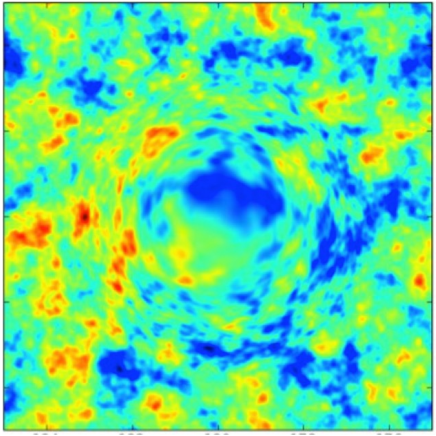

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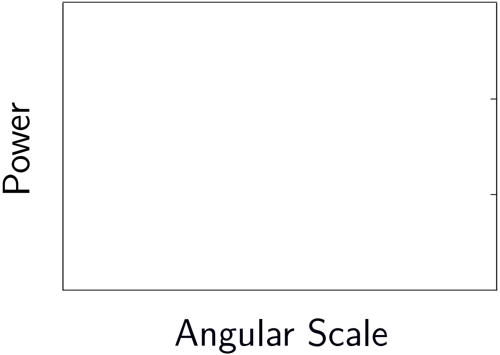

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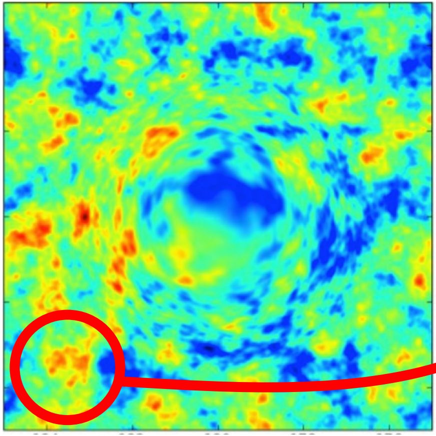

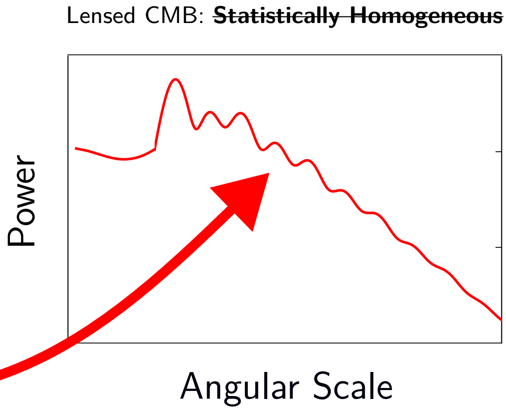

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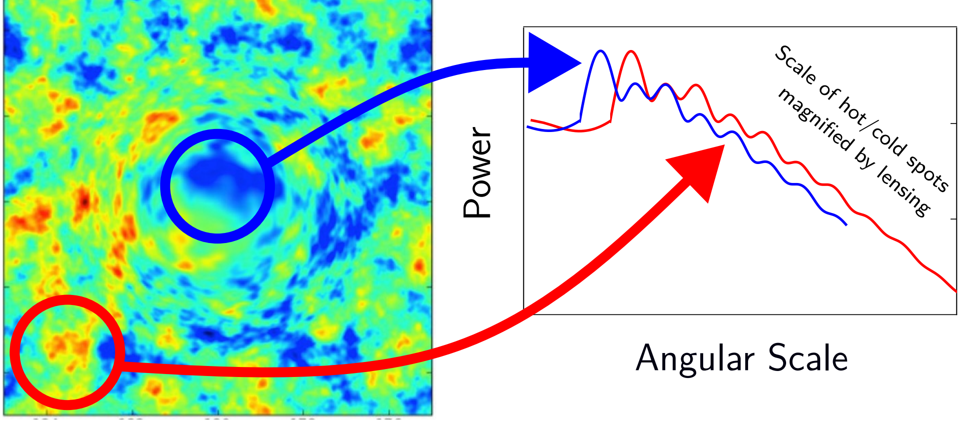

For a statistically **homogeneous** field like the unlensed CMB different Fourier modes are statistically independent:

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Lensing of the CMB breaks this symmetry by inducing correlations in our lensed CMB:

$$\boxed{\langle T_{\boldsymbol{\ell}} T_{\mathbf{L}-\boldsymbol{\ell}} \rangle \sim \kappa_{\mathbf{L}}}$$

$(\kappa \equiv -\nabla^2(\text{Lensing Potential})/2)$

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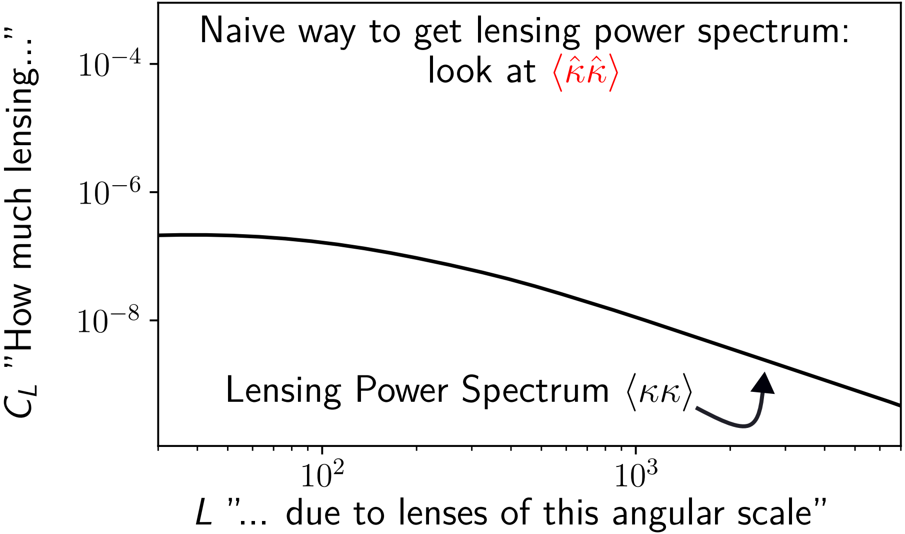
So correlations that we do see in our map give us information about the lensing allowing us to build an **quadratic estimator** (QE) of κ out of these correlations.

$$\boxed{\hat{\kappa}_{\boldsymbol{L}} \sim \int_{\boldsymbol{\ell}} T_{\boldsymbol{\ell}} T_{\boldsymbol{L}-\boldsymbol{\ell}}}$$



Naive way to get lensing power spectrum:
look at $\langle\hat{\kappa}\hat{\kappa}\rangle$
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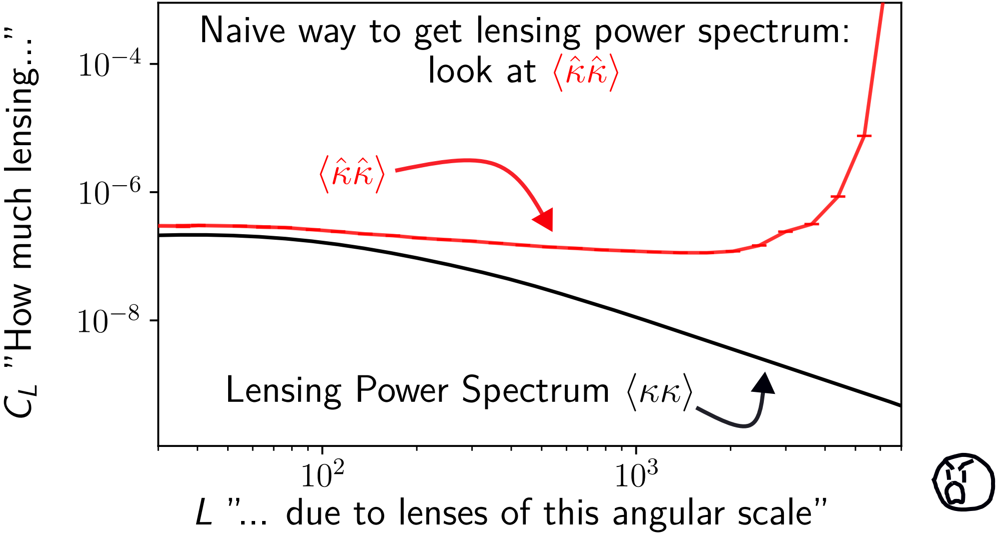

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$$\langle TTTT \rangle \sim \underbrace{\text{Large Gaussian Contribution}}_{\text{Present even if no lensing}} + \text{Lensing Term} + \ldots$$

$$\Rightarrow \langle \hat{\kappa}\hat{\kappa} \rangle \sim \text{Gaussian Bias} + \text{Lensing Power Spectrum} + \ldots$$

Furthermore, there's extra stuff in our temperature maps

1. Foregrounds (CIB, SZ, ...)
2. Detector noise **(for this talk, will focus on this)**

That are added to our maps:

$$T_\ell = T_\ell^{\mathrm{Lensed}} + N_\ell^{\mathrm{Detector}}$$

[1] Generically N^{Detector} is a inhomogeneous non-Gaussian field so has contributions beyond the "Gaussian noise", but for upcoming wide-field CMB maps, this detector noise can be expanded around a homogeneous Gaussian field.

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This changes $\langle TTTT \rangle$ by also contributing to the Gaussian bias[1]:

$$\Rightarrow \langle \hat{\kappa}\hat{\kappa} \rangle \sim \mathrm{Noise\ Bias} + \mathrm{Lensing\ Power\ Spectrum} + \ldots$$

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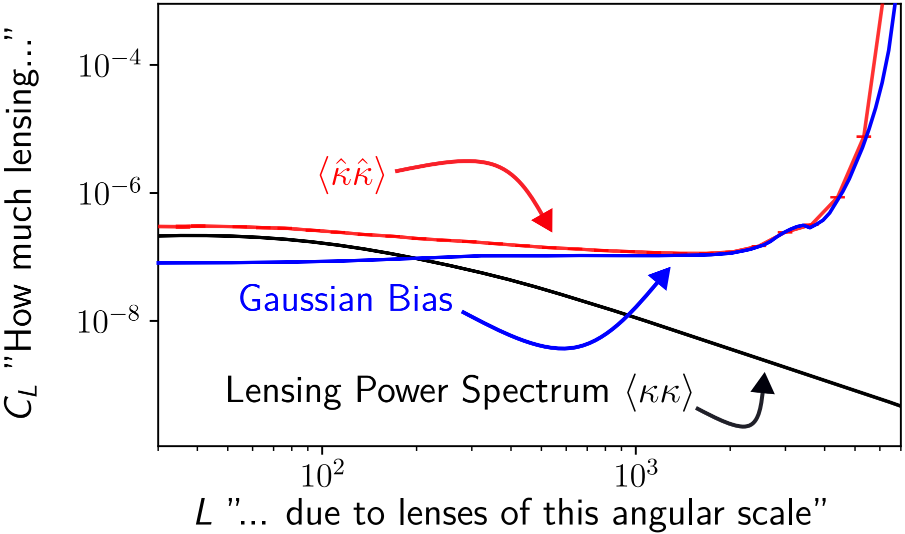

On small scales, even a tiny misestimate of Gaussian bias leads to a huge bias in estimated $\langle\kappa\kappa\rangle$

A key challenge in measuring the CMB lensing spectrum is estimating and subsequently **removing this Gaussian bias**[2].

[2] Stated more generally, we are trying to extract the non-Gaussian component of the 4-point function. This is a problem that appears generally in cosmology and the method we propose in principle is applicable to other areas where optimal estimation of the connected trispectrum/4-point function is of interest such as large scale structure.

A quick reminder: For a 1-D Gaussian random variable X with mean 0 and variance σ^2

$$\langle X \rangle = 0 \quad \langle X^2 \rangle = \sigma^2$$

1D analogue of power spectrum

$$\langle X^4 \rangle = 3\sigma^4$$

A toy model for estimating CMB lensing spectra is estimating $\mathcal{K} \ll \sigma^4$ for a **nearly** Gaussian variable X

$$\langle X \rangle = 0 \quad \langle X^2 \rangle = \sigma^2$$

$$\langle X^4 \rangle = 3\sigma^4 + \mathcal{K}$$

1D analogue of power spectrum and CMB lensing spectrum

"$\langle TTTT \rangle$"

" Noise Bias"

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Features: **(1)** noisy relative to minimum variance estimator and **(2)** maximally sensitivty to theoretical mismodelling

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Features: **(1)** is the minimum variance estimator but **(2)** still has some sensitvity to theoretical mismodelling

The standard way CMB Lensing Spectrum estimation is done "RDN(0)"

The **optimal** way to remove to remove this Gaussian bias for $\langle X^4 \rangle$

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Limitations of the Standard $\mathrm{RDN}^{(0)}$

- Relies on **computationaly expensive simulations**
- Estimated noise bias still **sensitive to small errors simulation**
  - **Small error** in noise bias is still **large error** in CMB lensing power spectrum
- In ACT DR6 (Qu+23), they could not simulate the complex instrument noise well enough to have unbiased measurements of the lensing spectrum at $L \sim 800$

Origin of the Gaussian bias

Recall that

$$\langle \hat{\kappa}_{\boldsymbol{L}} \hat{\kappa}^*_{\boldsymbol{L}} \rangle \sim \iint_{\boldsymbol{\ell},\boldsymbol{\ell}'} \langle T_{\boldsymbol{\ell}} T_{\boldsymbol{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}'} T_{-\boldsymbol{L}+\boldsymbol{\ell}'} \rangle$$

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- So in the integral, the Gaussian bias contributions come from terms where

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- Thus, by Eq. $(\star)$, the Gaussian bias comes from terms in $\int_{\boldsymbol{\ell}, \boldsymbol{\ell}'}$ where $\boxed{\boldsymbol{\ell} = \boldsymbol{\ell}'}$

Our Method: Ignore $\boldsymbol{\ell} = \boldsymbol{\ell}'$ terms

$$\text{Our estimator of } \langle\kappa\kappa\rangle \sim \iint_{\boldsymbol{\ell}\neq\boldsymbol{\ell}'} \langle T_{\boldsymbol{\ell}} T_{\mathbf{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}'} T_{-\mathbf{L}+\boldsymbol{\ell}'} \rangle$$

$$\text{(for practical purposes)} \sim \underbrace{\iint_{\boldsymbol{\ell},\boldsymbol{\ell}'} \langle T_{\boldsymbol{\ell}} T_{\mathbf{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}'} T_{-\mathbf{L}+\boldsymbol{\ell}'} \rangle}_{\text{The standard } \langle\hat{\kappa}\hat{\kappa}\rangle} - \underbrace{\int_{\boldsymbol{\ell}} \langle T_{\boldsymbol{\ell}} T_{\mathbf{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}} T_{-\mathbf{L}+\boldsymbol{\ell}} \rangle}_{\boldsymbol{\ell}=\boldsymbol{\ell}' \text{ terms}}$$

Both components can be computed efficiently using FFT!

Our Method: Ignore $\ell = \ell'$ terms

Our estimator of $\langle \kappa\kappa \rangle \sim \int\int_{\ell,\ell'} \langle T_{\ell} T_{L-\ell} T_{-\ell'} T_{-L+\ell'} \rangle$

$$\text{(for practical purposes)} \sim \underbrace{\int\int_{\ell,\ell'} \langle T_{\ell} T_{L-\ell} T_{-\ell'} T_{-L+\ell'} \rangle}_{\text{The standard } \langle\kappa\kappa\rangle} - \underbrace{\int_{\ell} \langle T_{\ell} T_{L-\ell} T_{-\ell} T_{-L+\ell} \rangle}_{\ell=\ell' \text{ terms}}$$

Both components can be computed efficiently using FFT!

Essentially we avoid the Gaussian bias by throwing away a small subset of our data

Features of our method

- Relies **only on data** thus
  1. Very fast to compute relative to standard $\mathrm{RDN}^{(0)}$
  2. Completely insensitive to errors in simulations

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  1. Very fast to compute relative to standard $\mathrm{RDN}^{(0)}$
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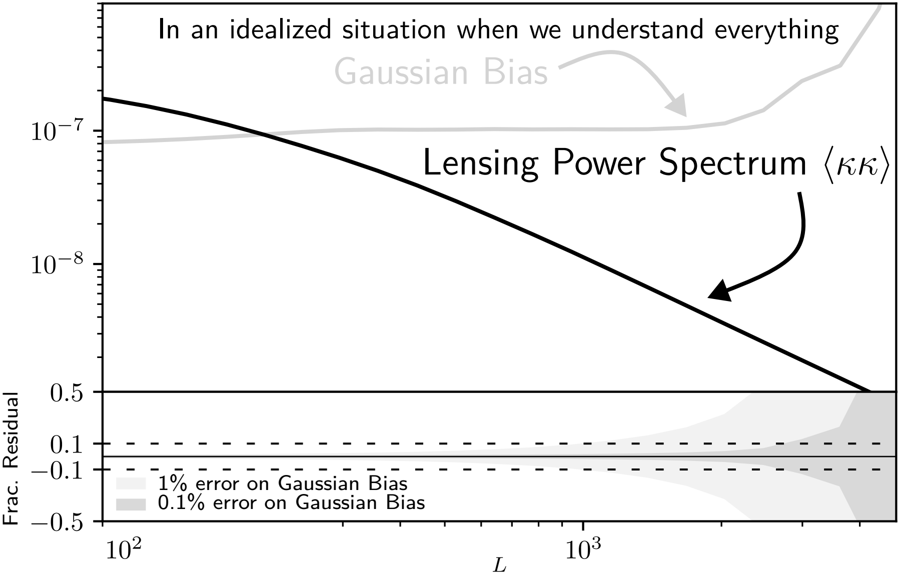
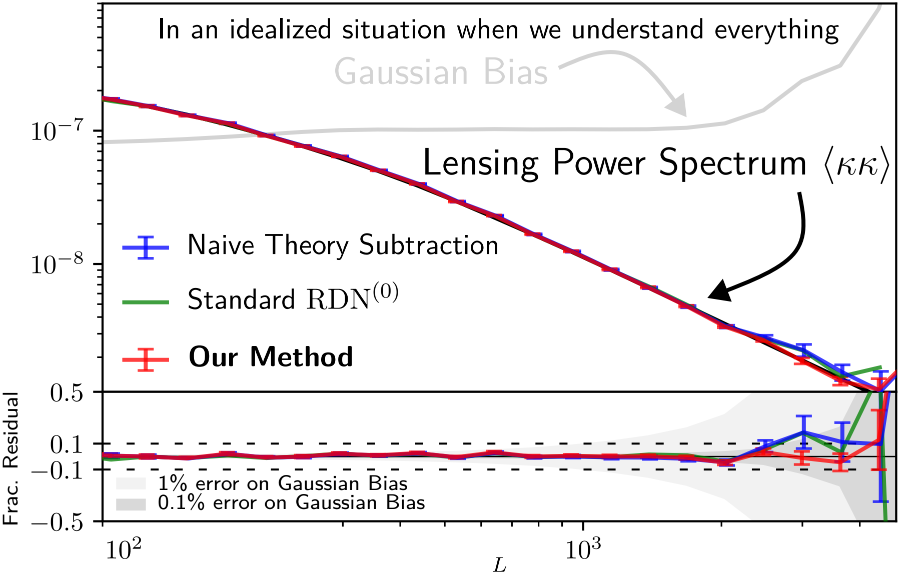
- One can reason using the toy model that the variance of **our method** is **asymptotically equivalent** to the variance of the **the optimal minimum variance estimator.**


In an idealized situation when we understand everything
Gaussian Bias
Lensing Power Spectrum $\langle\kappa\kappa\rangle$
10^{-7}
10^{-8}
0.5
0.1
−0.1
−0.5
Frac. Residual
1% error on Gaussian Bias
0.1% error on Gaussian Bias
10^2
10^3
L


In an idealized situation when we understand everything
Gaussian Bias
Lensing Power Spectrum $\langle \kappa\kappa \rangle$
Naive Theory Subtraction
Standard $\mathrm{RDN}^{(0)}$
Our Method
10^{-7}
10^{-8}
0.5
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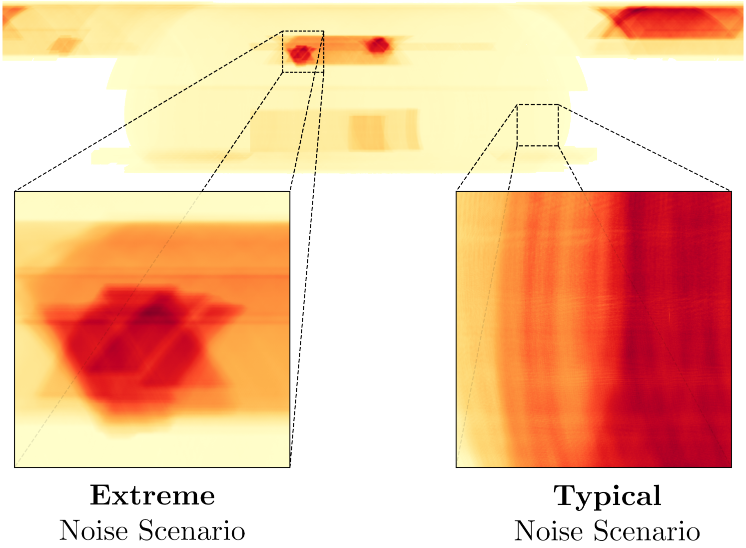
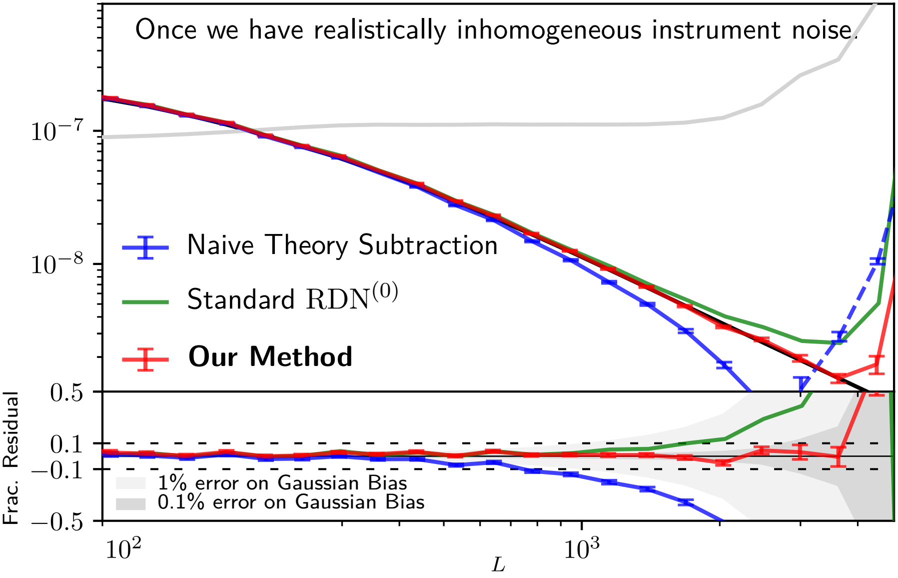
−0.1
−0.5
Frac. Residual
1% error on Gaussian Bias
0.1% error on Gaussian Bias
10^2
10^3
L

ACT DR5 IVar Map

On real data, a key limitation for standard methods is modeling complex instrument noise sufficiently accurately.




Once we have realistically inhomogeneous instrument noise.
Naive Theory Subtraction
Standard RDN$^{(0)}$
Our Method
1% error on Gaussian Bias
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Frac. Residual
10^{-7}
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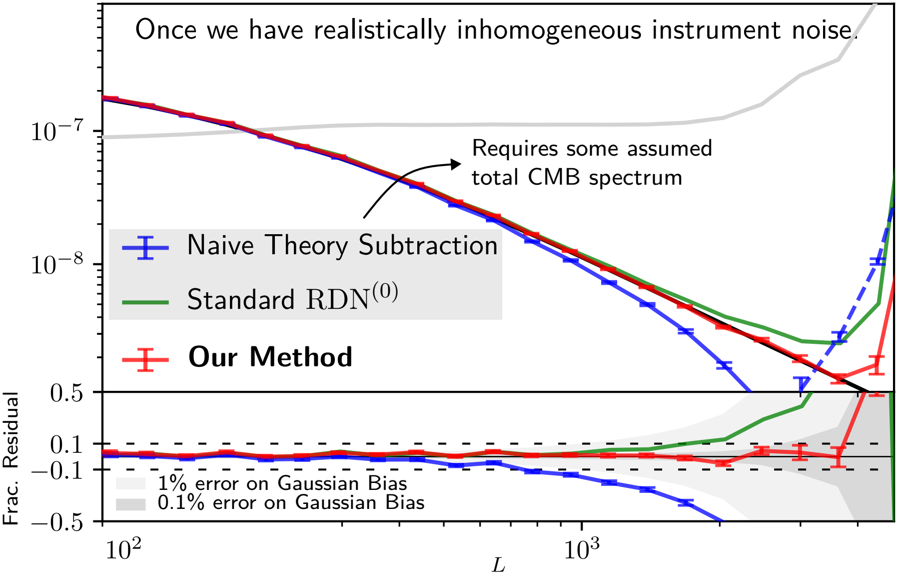

Once we have realistically inhomogeneous instrument noise.
Requires some assumed total CMB spectrum
Naive Theory Subtraction
Standard $\mathrm{RDN}^{(0)}$
Our Method
1% error on Gaussian Bias
0.1% error on Gaussian Bias
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10^3
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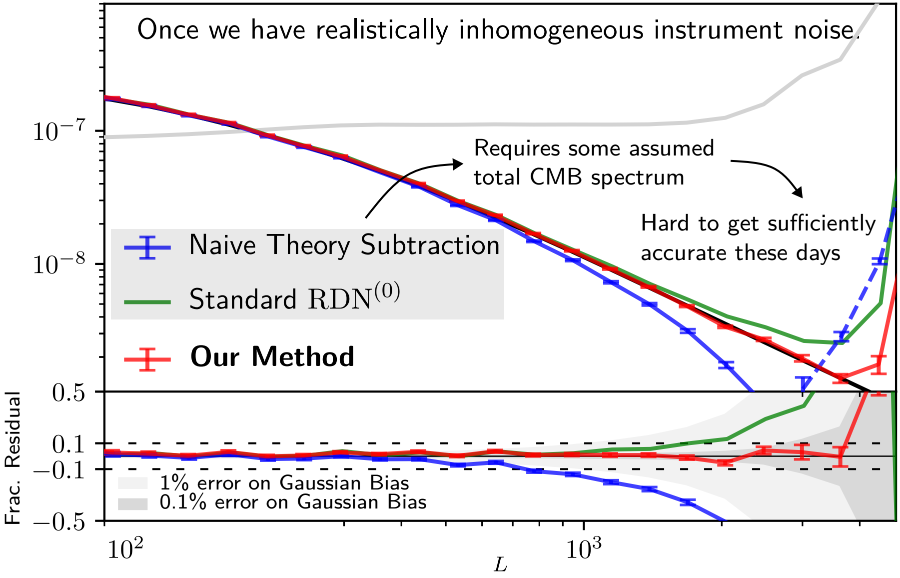
Once we have realistically inhomogeneous instrument noise.
Requires some assumed total CMB spectrum
Hard to get sufficiently accurate these days
Naive Theory Subtraction
Standard $\mathrm{RDN}^{(0)}$
Our Method
10^{-7}
10^{-8}
0.5
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Frac. Residual
1% error on Gaussian Bias
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10^2
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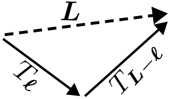
Conclusion

- We propose a novel estimator of the CMB lensing power spectrum which **relies only on data** making this estimator
  1. Fast to compute
  2. Insensitive to errors in simulations and thus unbiased to small scales
- This estimator's noisiness is **asymptotically equivalent to the optimal estimator.**
- We showed this estimator is robust to the presence realistic complications like inhomogeneous instrument noise
  - Can also show our estimator is as robust to masking as standard methods, happy to talk about this if people interested!

Extra

Our Method

Recall $\hat{\kappa}_{\boldsymbol{L}} \sim \int_{\boldsymbol{\ell}} T_{\boldsymbol{\ell}} T_{\boldsymbol{L}-\boldsymbol{\ell}} \sim \sum_{\boldsymbol{\ell}}$ 

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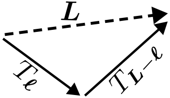
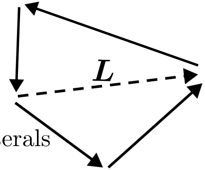


$$\langle \hat{\kappa}_{\boldsymbol{L}} \hat{\kappa}^*_{\boldsymbol{L}} \rangle \sim \sum_{\text{Quadrilaterals}}$$



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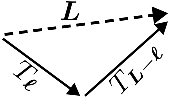

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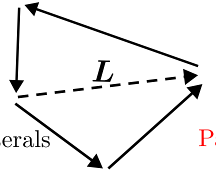
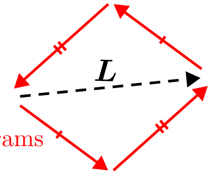
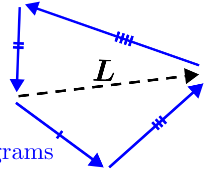


$\sim \sum_{\text{Parallelograms}}$



$+ \sum_{\text{Non-Parallelograms}}$



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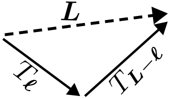

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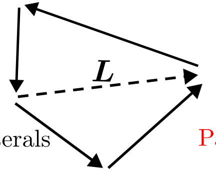

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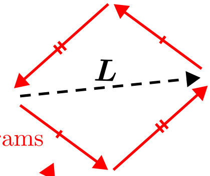

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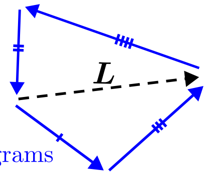

*(Contains contributions to lensing spectrum **and** Gaussian bias)*

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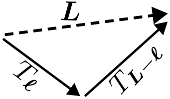

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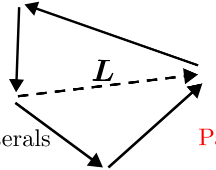
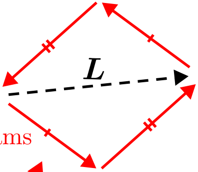
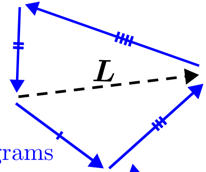


$\sim \sum_{\text{Parallelograms}}$



$+ \sum_{\text{Non-Parallelograms}}$



*(Contains contributions to lensing spectrum **and** Gaussian bias)*

*(Contains contributions to lensing spectrum **only**)*

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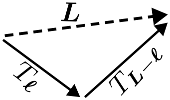

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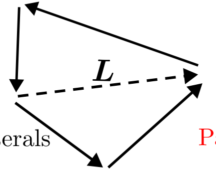

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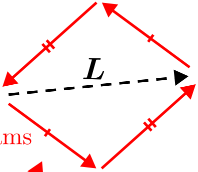

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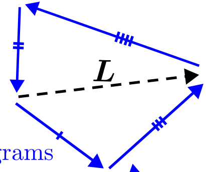
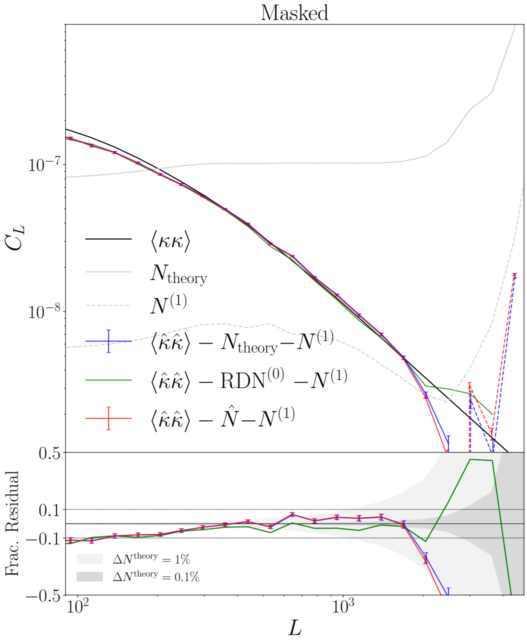


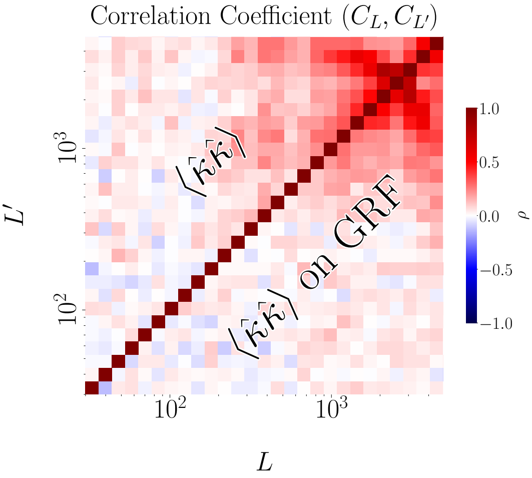
*(Contains contributions to lensing spectrum **and** Gaussian bias)*

*(Contains contributions to lensing spectrum **only**)*

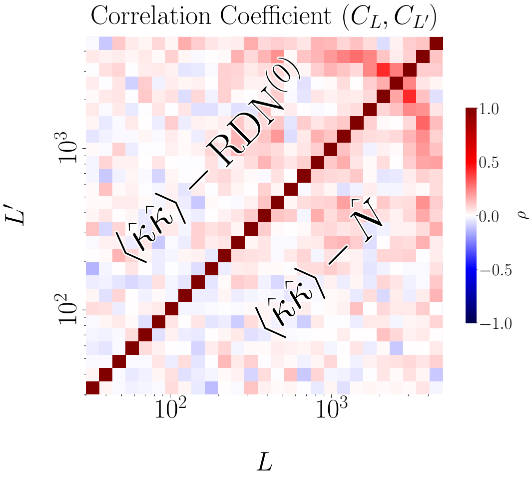
Idea: ignore all the quadrilaterals that **do** contribute to the Gaussian bias, the parallelograms

Masking introduces additional mode couplings and a mean-field that have to be handled. Here we show that our method is as good as the standard method in presence of masking! So usual methods to handle additional mode couplings should still work.

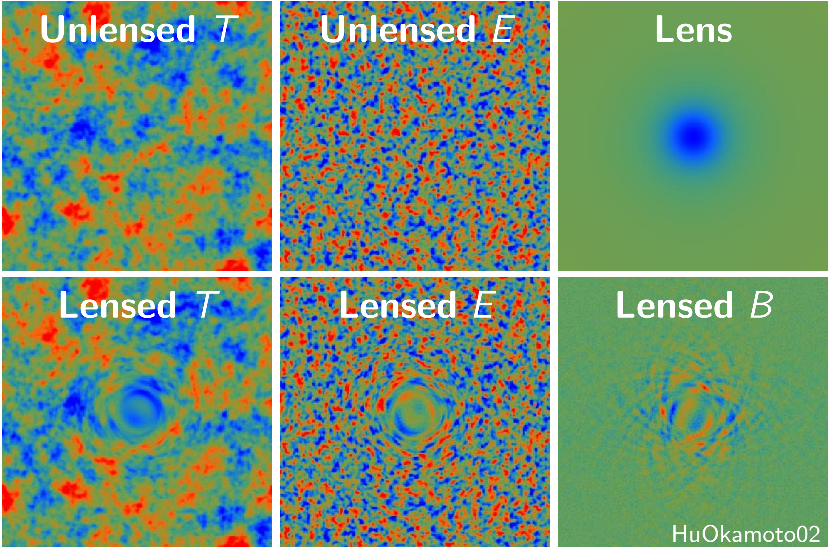




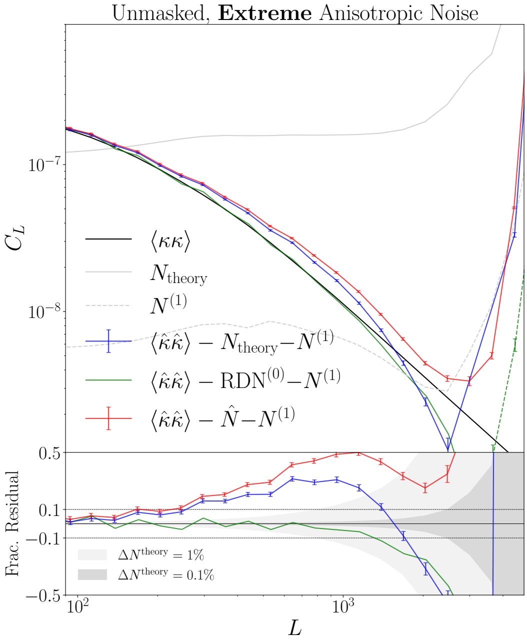
Non-trivial correlation structure due to Gaussian bias



Methods to remove Gaussian bias should also remove non-trivial correlation structure


Unlensed T
Unlensed E
Lens
Lensed T
Lensed E
Lensed B
HuOkamoto02

In our method we implicitly assume that noise inhomogeneities can be expanded perturbatively around a homogeneous limit. However, once this is not true, for example in the extreme inhomogeneous noise scenario, our method breaks down while the standard $\mathrm{RDN}^{(0)}$, which does not make this assumption, still works. However **we do expect the assumption that noise inhomogenities are small to hold for current and upcoming wide-field CMB maps.**



To intuitively understand the properties of (1) the current method and (2) the method we will propose to remove the Gaussian bias, lets consider a **toy model** for optimal trispectrum estimation[3].

---

[3] Originally presented in Smith+18

In Toy Model

Consider a **generic** weakly non-Gaussian random **variable** X with

- Zero mean
- Some assumed **variance** σ^2
- Small **kurtosis** $\mathcal{K} \ll \sigma^4$ we wish to estimate

$$\langle X^2 \rangle = \sigma^2$$

$$\langle X^4 \rangle = 3\sigma^4 + \mathcal{K}$$

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In CMB Lensing

Consider the **lensed CMB temperature map**, a weakly non-Gaussian random **field** with

- $\langle T \rangle = 0$
- Some assumed **power spectrum**
- Small **connected trispectrum** (induced by lensing) we wish to estimate

Let $\{x_i\}$ be N independent realizations of the random variable X. Generically, an estimator of the Kurtosis $\mathcal{K}$ will look like

$$\hat{\mathcal{K}} = \frac{1}{N}\sum_{i=1}^{N} x_i^4 - \hat{\mathcal{G}} = \text{(Sample 4-pt function)} - \text{(Estimate of Gaussian Bias)}$$

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Estimate of Gaussian Bias	$N \times \mathrm{Var}(\hat{\mathcal{K}})$	$\sigma^2 = \sigma^2_{\text{true}} - \epsilon$	In CMB Lensing
$\hat{\mathcal{G}}_{\text{naive}} = 3\sigma^4$			

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$\hat{\mathcal{G}}_{\text{opt}} = \frac{6\sigma^2}{N}\sum_i x_i^2 - 3\sigma^4$	$24\sigma^8$	$\Delta\mathcal{K} \sim \epsilon^2$	Current Standard

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$\hat{\mathcal{G}}_{\mathrm{alt}} = \frac{3}{N(N-1)}\sum_{i\neq j} x_i^2 x_j^2$	$24\sigma^8 + O(1/N)$		

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In Toy Model

The alternative estimator for the kurtosis $\hat{\mathcal{K}}_{\rm alt}$ has equivalent performance to optimal **kurotsis** estimator for large N but is completely insensitive to mismodelling of **the variance** σ^2

In Toy Model

The alternative estimator for the kurtosis $\hat{\mathcal{K}}_{\mathrm{alt}}$ has equivalent performance to optimal **kurotsis** estimator for large N but is completely insensitive to mismodelling of **the variance** σ^2

In CMB Lensing

Our estimator for the CMB lensing power spectrum $\langle\kappa\kappa\rangle$ asymptotically has equivalent performance to the optimal **trispectrum** estimator but is completely insensitive to mismodelling of **the observed CMB temperature power spectrum.**

In Toy Model

$$\hat{\mathcal{K}}_{\rm alt} = \frac{1}{N}\sum_i x_i^4 - \frac{3}{N(N-1)}\sum_{i\neq j} x_i^2 x_j^2$$

Since x_i is statistically independent from x_j when $i \neq j$, the second term yields only the the disconnected Gaussian bias.

In Toy Model

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Since x_i is statistically independent from x_j when $i \neq j$, the second term yields only the the disconnected Gaussian bias.

In other words, instead of assuming some σ^2, you estimate it directly from data.

In Toy Model

$$\hat{\mathcal{K}}_{\rm alt} = \frac{1}{N}\sum_i x_i^4 - \frac{3}{N(N-1)}\sum_{i\neq j} x_i^2 x_j^2$$

Since x_i is statistically independent from x_j when $i \neq j$, the second term yields only the the disconnected Gaussian bias.

In other words, instead of assuming some σ^2, you estimate it directly from data.

In CMB Lensing

Recall that

$$\langle \hat{\kappa}_{\boldsymbol{L}} \hat{\kappa}^*_{\boldsymbol{L}} \rangle \sim \int_{\boldsymbol{\ell},\boldsymbol{\ell}'} \langle T_{\boldsymbol{\ell}} T_{\boldsymbol{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}'} T_{-\boldsymbol{L}+\boldsymbol{\ell}'} \rangle$$

Our propsed estimator generalization of the toy model's $\mathcal{K}_{\rm alt}$

$$\langle \kappa\kappa \rangle \sim \langle \hat{\kappa}\hat{\kappa} \rangle - \int_{\boldsymbol{\ell}} \langle T_{\boldsymbol{\ell}} T_{\boldsymbol{L}-\boldsymbol{\ell}} T_{-\boldsymbol{\ell}} T_{-\boldsymbol{L}+\boldsymbol{\ell}} \rangle$$

Similar to the toy model case, the second term contains all the combination of $(\boldsymbol{\ell}, \boldsymbol{\ell}')$ contributing to $\int_{\boldsymbol{\ell},\boldsymbol{\ell}'}$ which contain a disconnected Gaussian bias.

What people usually do to estimate the Gaussian bias

In Toy Model

$$\hat{\mathcal{G}}_{\mathrm{opt}} = \frac{6\sigma^2}{N} \sum_i x_i^2 - 3\sigma^4$$

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In CMB Lensing

Let $\hat{\kappa}^{ds}$ be the QE using data for one temperature map and simulations for the other. Similar for $\hat{\kappa}^{ss'}$

$$\begin{aligned}\mathrm{RDN}_L^{(0)} \equiv \Big\langle & C_L(\hat{\kappa}^{ds}, \hat{\kappa}^{ds}) + C_L(\hat{\kappa}^{ds}, \hat{\kappa}^{sd}) \\ & + C_L(\hat{\kappa}^{sd}, \hat{\kappa}^{ds}) + C_L(\hat{\kappa}^{sd}, \hat{\kappa}^{sd}) \\ & - \left(C_L(\hat{\kappa}^{ss'}, \hat{\kappa}^{ss'}) + C_L(\hat{\kappa}^{ss'}, \hat{\kappa}^{s's})\right)\Big\rangle_{s,s'}\end{aligned}$$

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For modern data, simulations (1) expensive and (2) require modelling everything to exquisite accuracy